Effect of Locality of Node Mobility on Epidemic Broadcasting in DTNs

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***Abstract*—In DTNs (Delay/Disruption-Tolerant Networks) composed of mobile nodes, when the node movement has spatial locality, the nodes repeatedly miss opportunities to forward messages to other nodes, thus lowering communication performance. In this paper, we analyze the effect of locality of node mobility on message dissemination speed in epidemic broadcasting. We represent the locality of node mobility using the positional distribution of nodes in a stationary state, and we present a method for deriving the ratio of infected nodes from the positional distribution. Based on the results of a numerical experiment where the positional distribution of nodes obeys a two-dimensional normal distribution, we show that the message dissemination speed is heavily restricted by the locality of node mobility. Moreover, we clarify that a heavy-tailed positional distribution leads to a low locality of node mobility and entails mostly unrestricted message dissemination speed.**

I. Introduction

DTNs (Delay/Disruption-Tolerant Networks), represented by MANET (Mobile Ad-Hoc NETwork), where nodes are sparse, have attracted considerable attention. DTNs realize communication under conditions where continuous end-to-end connection is not guaranteed, and they are expected to realize promising applications in networks in a disaster areas, military networks, inter-planetary networks, sensor networks, and so forth. Many algorithms for achieving efficient communication in DTNs composed of mobile devices have been proposed , and most of them compensate for the lack of connectivity by implementing store-and-carry message forwarding.

Among methods for one-to-all communication in DTNs composed of mobile nodes, epidemic broadcasting allows nodes carrying a message (infected nodes) to forward the message when they enter the communication range of other nodes. A message spreads among nodes forming the network since all infected nodes forward messages repeatedly. Various algorithms for epidemic broadcasting are proposed in previous works (refer to [1] and references therein), in which performance metrics such as message delivery time, coverage and number of duplicate messages are evaluated.

Generally, node mobility affects the communication performance of DTNs, where nodes communicate through epidemic broadcasting or other manner, and previous works have conducted various evaluations regarding the effects of node mobility. In [2, 1], message delivery time and throughput are compared using various mobility models, including ones imposing constraints on the mobility of nodes, as in the case of vehicles on a road, in addition to fundamental mobility models, such as the random waypoint mobility model, the random direction mobility model and the random walk mobility model. Moreover, [3, 4] evaluates the effect of the distribution of rectilinear moving distance on message delivery time, buffer utilization and throughput using a mobility model in which nodes repeatedly perform rectilinear motion and randomly change their direction. The effect of heterogeneity of node mobility on message delivery time is also explored in [5].

As shown in Fig. 1, when nodes are likely to move around a specific area in the field, their trajectories do not intersect, and messages are difficult to spread because the frequency of contact between nodes is limited. However, in performance evaluations in the previous works mentioned above, the areas in which nodes can move are assumed to be equal, and mobility models are limited to models where the stationary positional distributions of nodes are identical. In [2], where the movement of each node is assumed to be restricted to a grid, the stationary positional distribution is the same for all nodes since they move under the same restriction. Moreover, in the mobility models in [3, 4], nodes are uniformly distributed in the field at a steady state. When nodes forming a DTN are represented by humans, node movement is centered at a certain point in the field since it is assumed that humans tend to move around the base of their activity (e.g., their home or office). If we consider this characteristic of human mobility, models in which node movement distribution in the field is not restricted do not necessarily reflect all the characteristics of motion of mobile nodes. To our knowledge, thus far there has been no research focusing on models with local positional distributions of nodes for evaluating message dissemination speed in epidemic broadcasting, although [6] has derived a scaling law of asymptotic throughput for MANET. Therefore, it is important to evaluate message dissemination speed under environments in which node movement is restricted and nodes are distributed locally in the field.

In our study, we evaluate the effect of locality of node mobility on message dissemination speed in epidemic broadcasting. First, we express the intensity of locality of node mobility through the shape of the positional distribution of nodes and clarify the relationship between positional distribution and frequency of contact between nodes. In this way, we derive a weighted adjacency matrix whose

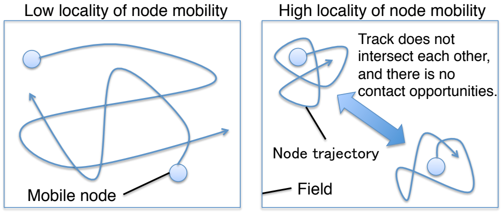

Fig. 1. Intensity of locality of node mobility and contact frequency.

elements are the frequencies of contact between each pair of nodes, and map the problem of epidemic broadcasting for mobile nodes to the problem of epidemics on a graph. In this paper, we compare message dissemination speeds for various intensities of the locality, where the positional distribution of nodes follow a two-dimensional normal distribution, by changing the standard deviation. Moreover, we examine the effect of the tail behavior of the positional distribution of nodes on message dissemination speed by comparing the results obtained with a two-dimensional normal distribution and a heavy-tailed two-dimensional Cauchy distribution.

The rest of the paper is organized as follows. First, in Section II, we present the analysis method used to evaluate the effect of locality of node mobility on message dissemination speed in DTNs employing epidemic broadcasting. Next, in Section III, we present the results of numerical experiments in which we compare message dissemination speeds for different intensities of the locality of node mobility. We conclude the paper and describe the direction of future work in Section IV.

II. Analysis

In the analytical model used in this paper, we express the intensity of locality of node mobility using the shape of the positional distribution of nodes. We assume that nodes $i\,(i = 1, 2, \ldots, N)$ have stationary motion patterns and therefore follow a stationary positional distribution. Note that we assume that the field in which the nodes can move is a two-dimensional space. The expression of locality of node mobility using positional distribution allows us to consider a wide variety of motion patterns. We quantify the intensity of locality of node mobility as the expectation of the distance L from the mean/mode point to the current node position. Larger $\mathrm{E}[L]$ indicates a wider range of motion of nodes and weaker locality of node mobility. Each node has a communication range r, and a node can communicate only when it is within the communication range of other nodes (in other words, only when nodes are in contact). An infected node forwards a message if it is in contact with a susceptible node. Note that we assume that infected nodes forward the message instantly, and we do not take message size into account.

To derive the message dissemination speed in epidemic broadcasting, first we derive the frequency of contact between nodes from the stationary positional distribution of the nodes. In DTNs, contact frequency strongly affects the performance of epidemic broadcasting since infected nodes forward and disseminate messages only when they are in contact with other nodes. Using the probability density function (pdf) of the positional distribution of node i, we can derive the total contact duration T_{total} per unit time for nodes i and j as follows.

$$\begin{aligned} T_{\text{total}} &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p_i(x,y) \int\int_{R_{x,y}} p_j(z,w)\mathrm{d}z\mathrm{d}w\mathrm{d}x\mathrm{d}y \\ &\simeq r^2\pi \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p_i(x,y)p_j(x,y)\mathrm{d}x\mathrm{d}y. \end{aligned} \tag{1}$$

where $R_{x,y}$ denotes the inside area of a circle with a center (x, y) and a radius r, and the approximate equation holds when r is sufficiently small. If we assume that contact duration is independent of contact frequency per unit time, we can derive the relation between the total contact duration T_{total} per unit time and the contact frequency $w_{i,j}$ for nodes i and j from the average contact duration T_{cd}.

$$w_{i,j} = \frac{T_{\text{total}}}{T_{\text{cd}}}. \tag{2}$$

Samar *et al.* have investigated the duration of contact between mobile nodes [7]. According to [7], we can obtain the average duration of contact $T'_{\text{cd}}(v)$ between a node with velocity v and other nodes as follows by assuming that the velocity of other nodes obeys a uniform distribution $U(0, v_{\max})$,

$$T'_{\text{cd}}(v) = \frac{r}{2v_{\max}}\int_0^{\pi} \log\left|\frac{v_{\max} + \sqrt{v_{\max}^2 - v^2\sin^2\phi}}{v + v\cos\phi}\right|\mathrm{d}\phi.$$

If we assume that v also obeys a uniform distribution $U(0, v_{\max})$, we can obtain T_{cd} as follows.

$$T_{\text{cd}} = \frac{1}{v_{\max}}\int_0^{v_{\max}} T'_{\text{cd}}(v)\mathrm{d}v. \tag{3}$$

As a result, we can derive the contact frequency for any arbitrary node pair by substituting Eqs. (1) and (3) into Eq. (2).

We can map the problem of epidemic broadcasting for mobile nodes with a stationary positional distribution $p_i(x, y)$ to the problem of epidemics on a graph G associated with a weighted adjacency matrix with elements $w_{i,j}$ since the rate at which an infected node i forwards a message to susceptible node j is given as the frequency of contact $w_{i,j}$ between nodes i and j. The weighted adjacency matrix of graph G is

$$A = \begin{pmatrix} 0 & w_{2,1} & \cdots & w_{N,1} \\ w_{1,2} & 0 & & w_{N,2} \\ \vdots & & \ddots & \vdots \\ w_{1,N} & w_{2,N} & \ldots & 0 \end{pmatrix},$$

where N denotes the number of nodes.

If the contact frequency is w for every node pair, the following differential equation holds by letting $I(t)$ denotes the ratio of infected nodes at time t [8].

$$\frac{\mathrm{d}}{\mathrm{d}t}I(t) = wI(t)(1 - I(t)). \tag{4}$$

Extending Eq. (4) by A, we can derive the following difference equation regarding the probability $\pi_i(t)$ that a node i is an infected node.

$$\boldsymbol{\pi}(t+\Delta t) = (E - \Pi)\, A\, \boldsymbol{\pi}(t)\Delta t + \boldsymbol{\pi}(t), \tag{5}$$

$$\boldsymbol{\pi}(t) = \begin{pmatrix} \pi_1(t) \\ \pi_2(t) \\ \vdots \\ \pi_N(t) \end{pmatrix}, \Pi = \begin{pmatrix} \pi_1(t) & 0 & \cdots & 0 \\ 0 & \pi_2(t) & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \ldots & \pi_N(t) \end{pmatrix}.$$

The ratio of infected nodes at time t is derived from $\boldsymbol{\pi}(t)$ as follows.

$$I(t) = \frac{1}{N}\sum_{i=1}^{N} \pi_i(t). \tag{6}$$

Therefore, by solving numerically the difference equation in Eq. (5), we can obtain the ratio of infected nodes $I(t)$ at time t, which indicates the message dissemination speed of epidemic broadcasting.

III. Numerical Experiment

Though our analysis method can apply to any positional distribution $p_i(x, y)$, we present numerical results in which the positional distribution follows a two-dimensional normal distribution as one of the simplest possible cases. We change the intensity of locality of node mobility by manipulating the standard deviation of the normal distribution, and compare the resulting message dissemination speed. We denote the pdf $p_i(x, y)$ of node i as

$$p_i(x, y) = \frac{1}{\sqrt{2\pi\sigma^2}} \mathrm{e}^{-\frac{(x-h_{x,i})^2}{2\sigma^2}} \cdot \frac{1}{\sqrt{2\pi\sigma^2}} \mathrm{e}^{-\frac{(y-h_{y,i})^2}{2\sigma^2}}, \tag{7}$$

where σ and $(h_{x,i}, h_{y,i})$ denote the standard deviation and coordinates of the mean point of the normal distribution, respectively. A smaller standard deviation σ leads to stronger locality of node mobility since the node movements are confined to a smaller area, and in this case the expectation $\mathrm{E}[L]$ of the distance L from $(h_{x,i}, h_{y,i})$ to the current node position is smaller. When the positional distribution of node i is given by Eq. (7), we can derive the total duration of contact T_{total} between nodes i and j as follows.

$$T_{\mathrm{total}} = \frac{r^2}{4\sigma^2} \mathrm{e}^{-\frac{(h_{x,i}-h_{x,j})^2+(h_{y,i}-h_{y,j})^2}{4\sigma^2}}$$

We consider the ratio of infected nodes at time t in a DTN composed of $N (= n \times n)$ nodes with the condition that the mean points $(h_{x,i}, h_{y,i})$ line up along n lines and n rows separated by a distance d which form a two-dimensional lattice as shown in Fig. 2. We denote the node infected at time 0 as node 0, and thereby $\boldsymbol{\pi}(0) = (1, 0, \ldots, 0)$.

We change the intensity of locality of node mobility by setting the standard deviation σ to 100 [m], 200 [m], 400 [m], 800 [m], 1600 [m], 3200 [m] and 6400 [m], and calculate the time until 50% of the nodes have become infected (50% delivery time) using Eqs. (5) and (6). We show the results in Fig. 3. The remaining parameters are $n = 10$, $d = 400$ [m], $r = 50$ [m] and $v_{\max} = 8000$ [m/h]. Figure 3 presents the results of a Monte Carlo simulation

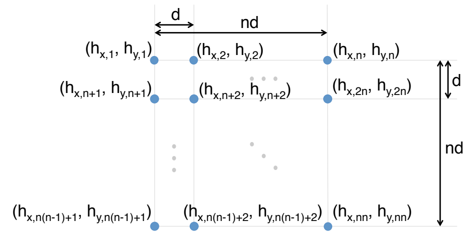

Fig. 2. Location of the mean points $(h_{x,i}, h_{y,i})$ of positional distributions of nodes

in which an infected node forwards a message randomly in accordance with the rate of link weights on a graph with a weighted adjacency matrix A. According to the figure, we can confirm that the 50% delivery time is extremely long in the case of $\sigma = 100$ [m]. The main reason for this is that infected nodes not likely to move to areas in which other nodes are distributed since the locality of node mobility is too strong. Similarly, the 50% delivery time is also long when σ is larger than 1600 [m]. The reason for this is the extremely weak locality of node mobility, as a result of which each node covers a rather wide area, and thereby many opportunities for establishing contact are lost.

Moreover, to investigate the effect of a heavy-tailed positional distribution of nodes on message dissemination speed, we compare the results for a two-dimensional Cauchy distribution with those for a two-dimensional normal distribution. The tail behavior of the positional distribution reflecting the motion patterns of nodes strongly affects the expectation $\mathrm{E}[L]$ of the distance L from a mean/mode point to the current node position. The Cauchy distribution is well known as a heavy-tailed distribution [9], although its pdf is symmetric and bell-shaped, similarly to the pdf of a normal distribution. Similarly to case of normal distribution, we give the positional distribution $p_i(x, y)$ of node i as follows.

$$p_i(x, y) = \frac{\gamma}{\pi((x-h_{x,i})^2+\gamma^2)} \frac{\gamma}{\pi((y-h_{y,i})^2+\gamma^2)}. \tag{8}$$

Note that γ and $(h_{x,i}, h_{y,i})$ denote the scale parameter and the coordinates of the mode point, respectively. When the positional distribution of node i is given by Eq. (8), we derive the total duration of contact T_{total} per unit time between nodes i and j as follows.

$$T_{\mathrm{total}} = \frac{4r^2\gamma^2}{\pi((h_{x,i}-h_{x,j})^2+4\gamma^2)((h_{y,i}-h_{y,j})^2+4\gamma^2)}.$$

Figure. 4 shows a comparison of the 50% delivery time for a Cauchy distribution (calculated by numerically solving the difference equation in Eq. (5)) and that for a Monte Carlo simulation. To perform direct comparison with the results for a normal distribution, we adjust the scale parameter γ such that x_c and x_n, satisfying $F_c(x_c) = 0.9$ and $F_n(x_n) = 0.9$, take the same value, where $F_c(x)$ and $F_n(x)$ denote the cumulative distribution functions of Cauchy and normal distributions. Therefore, the area in which a node is distributed with probability 0.8 (in other words, excluding the portions of the two tails of the distribution corresponding to a probability of 0.1 each) is the same for both distributions. Note that the horizontal

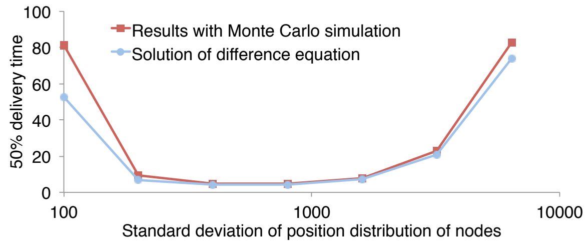

Fig. 3. 50% delivery times for different two-dimensional normal distributions used for the positional distributions of nodes.

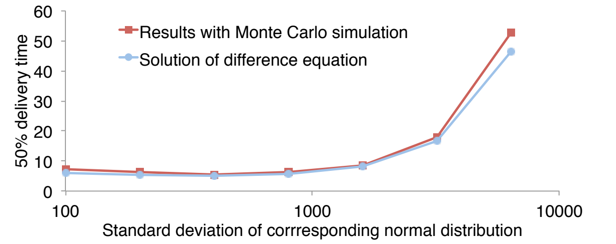

Fig. 4. 50% delivery times when the positional distributions of nodes follow a two-dimensional Cauchy distribution.

axis represents the standard deviation of the corresponding normal distribution, and other parameters are the same as in the example of a normal distribution. By comparing Fig. 3 with Fig. 4, we can confirm that although the 50% delivery time is long in the results of normal distribution when $\sigma = 100$ [m], the corresponding result for a Cauchy distribution is nearly the same as the results for Cauchy distribution that are corresponding to $\sigma = 200$ [m] and $\sigma = 400$ [m]. The main reason for this result is that compared with case of a normal distribution, in the case of a Cauchy distribution there is a high probability that a node contacts another node located far from it since the latter distribution has a heavy tail and the locality of node mobility is extremely weak.

The results for the examples mentioned above provide helpful suggestions for designing DTNs. In our result when $\sigma = 100$, heavy-tail of positional distribution reduces the 50% delivery time to about 1/10. For a heavy-tailed positional distribution of mobile nodes, we can expect that messages can be disseminated within a reasonable period of time, even if the density of nodes is very low and the nodes are located far apart. For instance, regarding real-world human movement patterns, the distribution of the distances to destinations is heavy-tailed [4], which suggests that the positional distribution of nodes is heavy-tailed. Moreover, in scenarios where we can control the motion patterns of nodes (e.g., in sensor networks), fast delivery can be achieved by designing heavy-tailed positional distributions.

IV. Conclusion and Future Work

We evaluated the effect of locality of node mobility on message dissemination speed in epidemic broadcasting. We derived the contact frequency between node pairs from the stationary positional distributions of nodes, and presented a method for deriving the ratio of infected nodes. Through numerical experiments, we investigated the case in which the positional distributions of nodes follow a two-dimensional normal distribution, and showed that the message dissemination speed is considerably lower when the standard deviation of the normal distribution is small and the locality of node mobility is strong. Moreover, we compared the results for a two-dimensional normal distribution and those for a heavy-tailed two-dimensional Cauchy distribution, and showed that message dissemination speed is almost unrestricted when the stationary positional distribution of nodes is heavy-tailed (entailing weak locality of node mobility). These results suggest that when the motion patterns of nodes follow a heavy-tailed positional distribution, messages can be disseminated within a reasonable period of time even if the density of nodes is low and they are located far from each other.

Although in this paper we focused on a method for deriving the ratio of infected nodes, a weighted adjacency matrix A whose elements are the frequencies of contact between nodes allows us to conduct various analyses. Epidemics on a graph have been studied in detail in a number of previous works. For instance, an analysis of epidemic thresholds has been conducted using the eigenvalues of an adjacency matrix . In future work, we plan to utilize these results to analyze message dissemination in the case of epidemic broadcasting in DTNs composed of mobile nodes.

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